TRINITY COLLEGE

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Section One: Calculator-free   **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1   **(7 marks)**

(a)   The distance run A_n km by an athlete in the nth week of training for an event is given by the recurrence relation $A_{n+1} = A_n + 5.5, \ A_1 = 16$.

   (i)   Use the relation to complete the following table.   (2 marks)

n	1	2	3	4	5
A_n					

   (ii)   What name is given to such a sequence of terms that model linear growth?
                                                                                (1 mark)

(b)   The distances run by another athlete in each of the first four weeks that they trained for the event form the sequence $15, 21, 27$ and 33 km.

   (i)   Deduce a rule for the nth term of this sequence of distances.   (2 marks)

   (ii)   In which week of training will this athlete first run at least 80 km? Justify your answer.   (2 marks)

See next page

Question 2   **(8 marks)**

Car owners who experienced a theft related to their car were asked which state they lived in and whether insurance covered the theft. The responses of 800 owners are shown below.

| State | Did insurance cover your theft? | | | Total |
	Yes	No, claim rejected	No, not insured	
Victoria	144	176		400
South Australia		100	75	
Western Australia	110		40	200

(a) Name the two categorical variables in the table.   (1 mark)

(b) Complete the two-way table above.   (2 marks)

(c) Use the theft data to complete the following row percentaged table.   (3 marks)

| State | Did insurance cover your theft? | | |
	Yes	No, claim rejected	No, not insured
Victoria			
South Australia			
Western Australia			

(d) Does the data suggest the presence of an association between the two variables? Justify your answer.   (2 marks)

Question 3   **(7 marks)**

An office is open five days a week (from Wednesday to Sunday inclusive), and the plot below shows the number of clients n visiting the office on each of 15 consecutive business days.
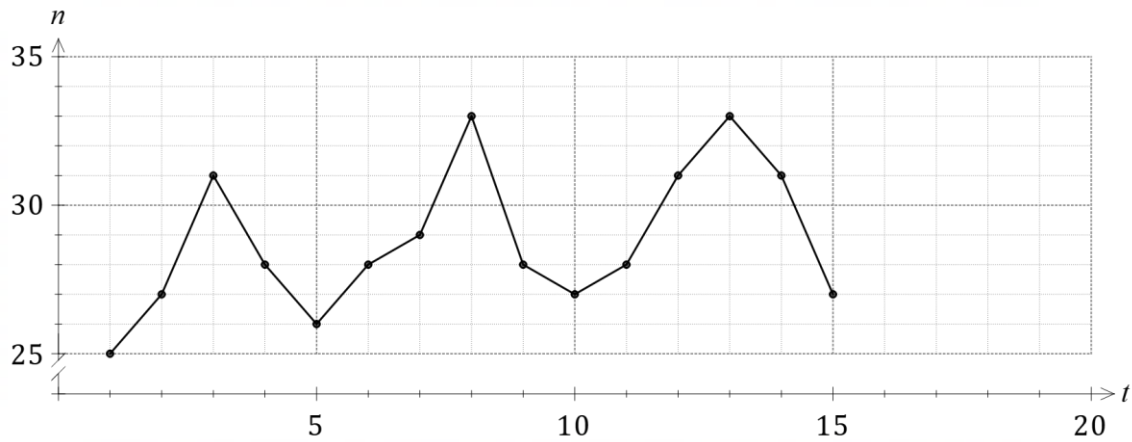
During the following week, the data below was recorded:

Day	Wed	Thu	Fri	Sat	Sun
Time, t	16	17	18	19	20
Number of clients, n	30	32	34	31	29

(a)   Use the data for the fourth week to complete the time series plot above.   (2 marks)

(b)   Describe the trend and seasonality of the time series.   (2 marks)

Using the average percentage method, the seasonal indices for Wednesday, Thursday, Friday and Sunday were calculated as $0.94, 1.01, 1.12$ and 0.93 respectively. The equation of the trend line for the number of clients visiting the office is $y = 0.2t + 26.9$.

(c)   Determine the seasonal index for Saturday.   (1 mark)

(d)   Write down a calculation that will best predict the number of clients expected to visit the office when $t = 23$. Do **not** evaluate your answer.   (2 marks)

Question 4   **(6 marks)**

The directed network below shows a system of conveyor belts used to move minerals from a receiving depot at X to a ship at Y. The edge weights indicate the tonnes of minerals that each conveyor belt can move per hour and the three dotted lines indicate cuts.
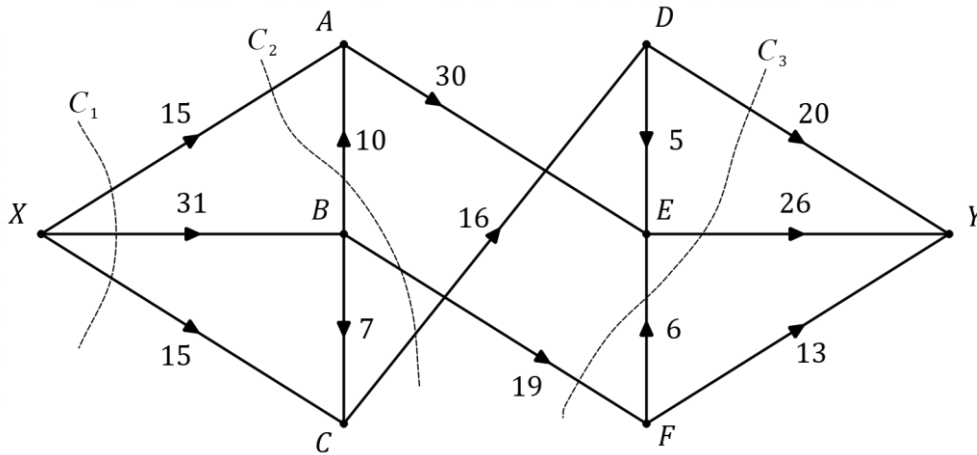
(a)   Determine the capacity of each of the three cuts shown above.   (2 marks)

(b)   Determine the maximum weight of minerals that can be moved from the receiving depot to the ship every hour.   (3 marks)

(c)   State the maximum possible increase to your answer in part (b) if the capacity of the conveyor belt from D to E could be increased by up to 4 tonnes per hour.   (1 mark)

See next page

Question 5   **(8 marks)**

(a)   In the bipartite graph shown below, one set of vertices represents swimmers, and the
       other set represents the events that they will compete in. If vertex 1 is an event, determine
       which of the vertices represent a swimmer.   (2 marks)
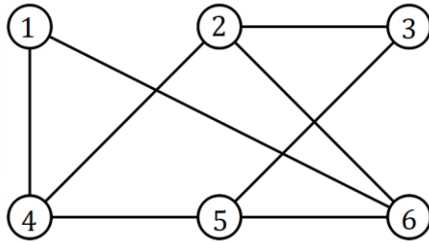
(b)   A connected planar graph has 15 vertices and 21 edges. Determine the number of faces
       the graph has.   (2 marks)

(c)   If possible, draw a simple connected graph that has 3 vertices and 4 edges. If not
   possible, explain why not.   (2 marks)

(d)   There are $\frac{n}{2}(n-1)$ edges in a complete graph with n vertices. Determine the number of
       edges that must be added to a tree with 10 vertices so that it becomes a complete graph.
                                                                                   (2 marks)

Question 6   **(9 marks)**

In the graph shown below, the vertices represent the eight branches of a bank in a city, the edges indicate that a courier service exists between adjacent branches and the weight on each edge is the number of sets of traffic lights that a courier travelling from one branch to another must pass through.
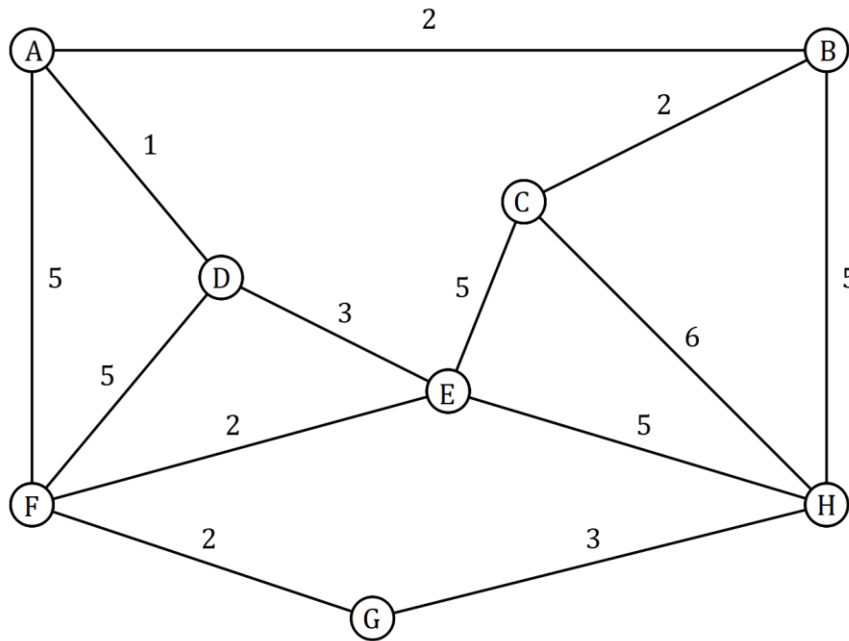
(a)   Determine the number of zeros in the adjacency matrix for the graph.   (2 marks)

(b)   Explain why the graph cannot be Eulerian.   (1 mark)

(c)   Explain why the graph is Hamiltonian.   (2 marks)

(d) A courier must collect a document from branch G, visit each of the other seven branches exactly once to get it signed and then return the document to branch G.

    (i) Determine two possible orders in which the courier could visit the branches.

(2 marks)

    (ii) Determine the least number of sets of traffic lights that the courier could pass through and state the corresponding order in which the branches are visited.

(2 marks)

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Question 7   **(7 marks)**

One Sunday evening, a catering company has to assign four food trucks, 1, 2, 3 and 4, to four locations, A, B, C and D. Each truck must be assigned to exactly one location for the entire evening. Truck 3 cannot be assigned to location D and truck 4 cannot be assigned to location C.

The table below shows the profit, in hundreds of dollars, that each truck is expected to make at each location.

	A	B	C	D
1	36	41	40	37
2	43	42	44	40
3	41	42	44	–
4	43	40	–	41

(a)   Use the Hungarian algorithm to determine an assignment of trucks to locations that will maximise the total profit made by the four trucks.   **(6 marks)**

(b)   State the resulting total profit.   **(1 mark)**

End of questions

Supplementary page

Question number: _____